INTERACTION DIAGRAM FOR UNI-AXIALLY LOADED SLENDER REINFORCED CONCRETE COLUMN

By

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Declaration

Eyobed Woldeab Teshome declares that this study is original and has not been published and/or submitted for any degree award to any other university or institution of higher learning before.

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Signature

This thesis has been submitted for examination with the approval of the following supervisor:

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APPROVAL PAGE

This MSc thesis titled "Interaction Diagram for Uni-Axially Loaded Slender Reinforced Concrete Column" has been approved by the following examiners in partial fulfillment of the requirement for the degree of Master of Science in Structural Engineering.

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CERTIFICATION

I, the undersigned, certify that I read and hereby recommend for acceptance by Addis Ababa Science and Technology University a dissertation titled "Interaction Diagram for Uni-Axially loaded slender reinforced concrete column" in partial fulfillment of the requirement for the degree of Master of Science in Structural Engineering.

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ABSTRACT

In the construction industry, a lot of advancements have been made in the past few decades. Column designing is among the advancements in general and slender columns in particular. Their aesthetical quality, the growing use of high strength materials and efficiency in use of space are some of the attributes that makes them suitable for building construction. The use of slender reinforced concrete columns is however restricted due to lack of local design aids, and there is a need for design information, particularly regarding interaction diagrams which are very helpful in the design and analysis of such members. In this study, slender uni-axial interaction diagram was the primary approach in solving the problem. A simple and user-friendly program was developed to determine the moment-load contours based on assumed curvature. The procedure is tested by a comparison with results provided by design software ETABS, which is a code based approach. Unfortunately, due to high variance when compared to the ETABS results, the approach had to be changed. A first order moment and axial load approach was used as an alternative using C++ program. The essential steps of the procedure include starting from geometric, reinforcement and material data (b,h,L,A_s,f_{ck},f_{yk}) the unit less load and moment data (μ_{input}& v_{input}) are obtained; relationships and equations on the normal interaction diagram are calculated; a comparison is made b/n (μ_{input}& v_{input}) and interaction diagram points; and iteration is carried out until convergence is achieved. The data from this program can aid in preparation of design chart for uni-axially loaded slender isolated RC column.

ACKNOWLEDGMENTS

Thanks God for giving me the urge to move forward and surpass each obstacle. I would like to thank my thesis advisor, Dr. Ing. Adil Zekaria, for his incisive comments that have been instrumental for looking into the design of slender column. He has been really stimulating and inspiring advisor. I thank the Addis Ababa Science and Technology University and the Ethiopian Roads Authority that provided financial support to accomplish my study. Finally, I am deeply indebted to three people without whose patience, understanding and encouragement this thesis would never have been completed: My parents W/ro Nishan Bekele and Dr. Woldeab Teshome and my sister Blen Woldeab.

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LIST OF SYMBOLS AND NOTATIONS

Latin uppercase letters

A_c Gross area of concrete section

ACI American Concrete Institute

A_{sN}= Area of reinforcement required to resist axial load

$A_{s,tot}$ Total area of reinforcement in columns

Cc Compressive force developed in the concrete.

Cs1 Tensile force or compression force developed in the bottom reinforcement of column cross section

Cs2 Compressive force developed in the in the compression reinforcement pg.

EBCS Ethiopian Building Code Standard

E_{cm} Compressive strain in the outer most fiber

Ec Tangent Modulus of Elasticity of Concrete at stress σ=0 and 28 days

Euro code European STANDARD

ETABS Extended Three Dimensional Analysis of Building Systems

Es Design value of modulus of elasticity of reinforcing steel

Fcd Design compressive strength of concrete.

Fck Characteristic compressive strength of concrete.

Fyd Design yield strength of reinforcement.

I Moment of inertia

Ig Gross moment of inertia of section about centroid

K Coefficient; factor

L Clear height of column

Le Effective buckling length

Mo First order moment of column in uni-axial bending

M_{0Ed} is the 1st order moment, including the effect of imperfections.

M_{01}, M_{02} are the first order end moments, $|M_{02}| \geq |M_{01}|$

M_2 is the nominal 2nd order moment.

M_{bal} Balanced moment capacity of column in uni-axial bending

M_{Ed} Design moment at the critical section including second order effect

Msc Slender column cross section ultimate moment capacity of column in uni- axial bending

Mu Ultimate moment capacity of column in uni-axial bending

N_{Ed} is the design value of axial load.

Nc Compressive force developed in the concrete.

P_{sd} Design values of internal axial load

P_{sc} Slender column axial load capacity

P_u Ultimate axial load capacity of column.

RC Reinforced concrete

SRCC Slender Reinforced Concrete Column

Latin lower case letters

b, h - Dimensions of rectangular section (width, height)respectively

d - Effective depth of rectangular section

e - Eccentricity

e_o Equivalent uniform first order eccentricity

e_a Additional eccentricity

e_2 Second order eccentricity

e_{tot} Total eccentricity

fcu Cube compression strength of concrete

h’ Concrete cover to the centroid of the reinforcement

i Radius of gyration

k_x Relative depth of neutral axis

l_0 Effective length of column

$n = Vu = \frac{Pu}{f_{cd}Ac}$, Design value of the ultimate relative axial load

r Radius of curvature

x Depth to neutral axis

Greek lower case letters

α_c Relative compressive force in concrete under compression

βc Relative distance of point of application of the compressive force in the concrete ,Cc from the outermost concrete fiber under compression.

λ Slenderness ratio

λ_{lim} limiting slenderness ratio

ε_o Strain at the point on the parabolic _rectangular stress diagram where the parabolic section joins the linear section.

ε_{sy} Strain reinforcement at the yield point

ε_{yd} Design of yield strain of steel

Φ Curvature.

ε_{s1} Strain in tensile reinforcement

ε_{s2} Strain in compressive reinforcement

ω Reinforcement ratio

ϕ_{ef} is the effective creep ratio

$\mu_u = \frac{Pu}{f_{cd}A_c h}$, Design moment capacity of columns with uni-axial bending

$\mu_{sd} = \frac{M_{Ed}}{f_{cd}A_c h}$, Design moment of columns with uni-axial bending respectively.

$Vsd = \frac{Psd}{f_{cd}Ac}$, Design value of the ultimate relative axial load

Table of Content

1. Introduction

1.1 Background

Fig.1: Historical building

In the construction industry, a lot of advancements have been made in the past few decades. Column designing is among the advancements in general and slender columns in particular. Their aesthetical quality, the growing use of high strength materials and efficiency in use of space are some of the attributes that makes them suitable for building construction. Slender columns can be found in lobby of buildings and lining exterior of buildings. A perfect example of this can be found in ancient roman architecture, the Maison Carrée for example has slender columns lining the exterior of the building(fig.1). *Duiker* and *Spielvogel* (2010:111*)* have noted that *"*The Greeks used different shapes and sizes in the columns of their temples. The Doric order, evolved first in the Dorian Peloponnesus, consisted of thick, fluted columns with simple capitals (the decorated tops of the columns). The Greeks considered the Doric order grave, dignified, and masculine. The ionic style was first developed in western Asia Minor and consisted of slender columns with spiral-shaped capitals. The Greeks characterized the Ionic order as slender, elegant, and feminine. Corinthian columns, with their more detailed capitals modeled after acanthus leaves, came later, near the end of the fifth century B.C.E."

1.2 Objectives

The general objective of the study is to prepare interaction chart for slender rectangular RC column subject to uniaxial loading.

The specific objectives of the study are:

a) To develop a computer program that generates deflection for rectangular RC column; and
b) To generate interaction diagram for slender uni-axial isolated column.

1.3 Statement of the problem

In the early 1960s, a study conducted by Macgregor, et al. reason for conducting the research "the main arguments for a revision have generally been based on the following short comings of the present reduction factor design method (keep in mind the research was focused on the 1963 ACI code):

1. The reduction factor method implies maintenance of the same eccentricity in both the slender and analogous short column. This is contradictory to the actual behavior of slender columns where the reduction in load-carrying capacity is caused by the increased eccentricity due to secondary deflection moments. This is a severe shortcoming in the case of unbraced frames, since the magnitude of the secondary moments is extremely important and should be included in the design of the restraining beams
2. Due to practical considerations, many important variables had to be neglected in trying to express the reduction factor method in a designer-

usable form. Because of this, the reduction factor expressions were based on extreme lower bounds and are unduly conservative for many practical cases.

3. The entire treatment of the slender column problem lacks rationality in ACI 318-63. Little encouragement or direction is given to the designer to use a more comprehensive method of analysis which considers the secondary moments and actual member response. In view of the rapidly developing capacity for improved structural analysis using computers, the design method for slender columns should actively encourage the use of a highly accurate second-order structural analysis wherever possible.

Therefore, the conclusion drawn was as follows. In place of such an analysis it proposed an approximate design method based on a moment magnifier principle and similar to the procedure used under the AISC specifications. It presents an outline of the normal range of variables in column design and proposes a lower limit of applicability which will eliminate over 90 percent of columns in braced frames and almost half of columns in unbraced frames from considerations as slender columns. Through a series of comparisons with analytical and test results, the accuracy of the approximate design procedure is established. It is shown that the proposed procedure is more rational, more accurate, and more consistent than the presently used procedure. Because the proposed method calls the attention of the designer to the basic phenomenon in slender columns and allows him to evaluate the additional moment requirements in restraining members, a superior and safer design results."

Another study by Eldon F. Mockery and David Darwin (slender column interaction diagrams) attempts to simplify the design time (computational savings) over the method outlined in the ACI design handbook. “The handbook procedure is an iterative process requiring initial assumptions for the reinforcing ratio, followed by successive trials. The slender column interaction method is a direct method, requiring no initial assumptions regarding the reinforcing ratio. The example calculations show that the proposed method provides results in a single solution using the method illustrated in the ACI Design handbook.” The conclusion follows “the interaction diagrams developed provide a direct solution for the reinforcing ratio of single columns. To use the diagrams, the designer does not have to make initial reinforcing ratio assumptions or iterative calculations. The approach presented here will reduce the design time for reinforced concrete structures.”

In literature, several researches attempted to introduce new approach to slender column design, and present a critical review for most of the recommended design methods in Codes and recent researches. Hyo-Gyoung Kwak introduces a new slender column design approach. This approach focused on developing improved design formula for bi-axially loaded slender reinforced concrete column. A simple but effective regression formula is proposed for the design of slender RC columns subjected to uni-axial or biaxial bending moments. The following conclusions were drawn from the results of this limited investigation:

(1) The results of the ACI method are in good agreement with the results of a rigorous analysis of RC columns with a relatively small slenderness ratio, regardless of the changes in design variables;

(2) The ACI method yields very conservative results when the slenderness ratio and the ultimate creep coefficient of concrete increase; and

(3) The results of the proposed formula are in good agreement with the results of a rigorous analysis when a consistent difference is maintained over the entire eccentricity for all slender RC columns.

Although rigorous numerical methods that consider material and geometric nonlinearities play an increasingly important role and will become the standard for final design checks, the formula proposed in this paper can be effectively used to determine the initial section of a slender RC column. Moreover, a more rational approach can be developed by conducting extensive studies, including experimental studies, on reliability assessment. Hamdy Elgohary proposed a new simple and direct design approach considering both geometric and material nonlinearity by assuming a deflection curve with a sinusoidal function. The conclusion of the research is as follows. The evaluation review of the current Codes' formulae and some recommended empirical approaches for the design of RC slender columns shows that most of these methods under -estimate and/or over-estimate the lateral deflection and/or the effective flexural rigidity. Based on this evaluation and considering the evaluation experimental results new formulae for the total lateral deflection calculation and effective flexural rigidity are recommended. In these formulae the effect of geometric nonlinearity and the material nonlinearity have been taken into consideration. The results obtained using the recommended equations show good agreement with the evaluation experimental results.

In this research, I aim to determine the deflection via curvature method. In doing so, I will not use the deflection approximation formulas outlined in the EBCS's code. Instead, I intend to determine deflection first by sectioning the column in to several equal parts, then determining the axial force and moment as well as curvature data at each section. This is a more general method, which promises to help bridge the knowledge gap and has not been addressed by the researches described earlier. Further, the study will attempt to generate interaction diagram for slender uni-axial isolated column.

1.4 Scope and limitation of the study

The study focuses on uniaxial loaded slender rectangular RC column. The study does not consider other types of cross sections such as circular column and triangular column. For this project, I considered columns with and without crack for a more realistic result. In addition, shrinkage, tension stiffening effects and effects of long term loading are not in scope for this study.

1.5 Organization of the thesis

The thesis has five chapters. The introductory chapter contains background, objectives, statement of the problem, scope and limitation of the study. The second chapter reviews different slender design approaches including curvature method and moment magnification method. The third chapter deals with the assumptions and the method used to conduct the study. The fourth chapter presents finding of the study. The fifth chapter provides the main conclusion and recommendations.

2. Literature review

2.1 Design approach[1]

For the design of columns the elastic moments from the frame action should be used without any redistribution. For slender columns a non-linear analysis may be carried out to determine the second order moments; alternatively use the moment magnification method or nominal curvature method.

Design moments

The design bending moment is defined as:

$M_{Ed} = \text{Max}\{M_{02}, M_{0e} + M_2, M_{01} + 0.5\ M_2\}$

(2.1)

Where

$M_{01} = \text{Min}\{|M_{top}|, |M_{bottom}|\} + e_i\ N_{Ed}$

(2.2)

$M_{02} = \text{Max}\{|M_{top}|, |M_{bottom}|\} + e_i\ N_{Ed}$

(2.3)

$$e_i = Max\left\{{}^{l_o}/_{400}, {}^{h}/_{30}, 20\right\} \quad \text{(Units to be in millimeters)} \qquad (2.4)$$

M_{top}, M_{bottom} = Moments at the top and bottom of the column

$M_{0e} = 0.6\ M_{02} + 0.4\ M_{01} \geq 0.4\ M_{02}$ (2.5)

[1] This review is taken form 'How to design concrete structures using Eurocode 2' 5. Columns by R Moss and O Brooker BEng, The Concrete Centre™ and British Cement Association, 2006

$M_2 = N_{Ed}\, e_2$ where N_{Ed} is the design axial load and e_2 is deflection due to second order effects (2.6)

M_{01} and M_{02} should be positive if they give tension on the same side.

A non-slender column can be designed ignoring second order effects and therefore the ultimate design moment, $M_{Ed} = M_{02}$.

The calculation of the eccentricity, e2, is not simple and is likely to require some iteration to determine the deflection at approximately mid-height.

The next section, e2 is determined in the curvature method

(3) The nominal second order moment M_2 in Expression (2.1) is

$M_2 = N_{Ed} e_2$

(2.7)

Where:

N_{Ed} is the design value of axial force

e_2 is the deflection = $(1/r)\, l_o{}^2/c$ (2.8)

$1/r$ is the curvature, (2.9)

l_o is the effective length, (2.10)

c is a factor depending on the curvature distribution, (2.11)

(4) For constant cross section, c= 10 ($\approx \pi^2$) is normally used. If the first order moment is constant, a lower value should be considered (8 is a lower limit, corresponding to constant total moment).

2.1.1 Curvature method

(1) For members with constant symmetrical cross sections (incl. reinforcement), the following may be used:

$$1/r = K_r K_\phi \, 1/r_0 \tag{2.12}$$

Where:

K_r is a correction factor depending on axial load, (2.13)

K_ϕ is a factor for taking account of creep, (2.14)

$$1/r_o = \varepsilon_{yd}/(0{,}45d) \tag{2.15}$$

$$\varepsilon_{yd} = f_{yd}/E_s \tag{2.16}$$

d is the effective depth; (2.17)

(2) If all reinforcement is not concentrated on opposite sides, but part of it is distributed parallel to the plane of bending, d is defined as

$$d = (h/2) + i_s \tag{2.18}$$

Where i_s is the radius of gyration of the total reinforcement area

(3) K_r in Expression (2.12) should be taken as:

$$K_r = (n_u - n)/(n_u - n_{bal}) \leq 1 \tag{2.19}$$

Where:

$n = N_{Ed}/(A_c f_{cd})$

(2.20)

N_{Ed} is the design value of axial force

$n_u = 1 + \omega$ (2.21)

n_{bal} is the value of n at maximum moment resistance; the value 0.4 may be used

$$\omega = A_s f_{yd}/(A_c f_{cd}) \tag{2.22}$$

A_s is the total area of reinforcement

A_c is the area of concrete cross section

(4) The effect of creep should be taken into account by the following factor:

$$K_\phi = 1 + \beta\phi_{ef} \geq 1 \tag{2.23}$$

Where:

ϕ_{ef} is the effective creep ratio, (2.24)

$$\beta = 0.35 + f_{ck}/200 - \lambda/150 \tag{2.25}$$

λ is the slenderness ratio, (2.26)

Effective length

Figure 3 gives guidance on the effective length of the column.

Fig 2. Design Bending Moments

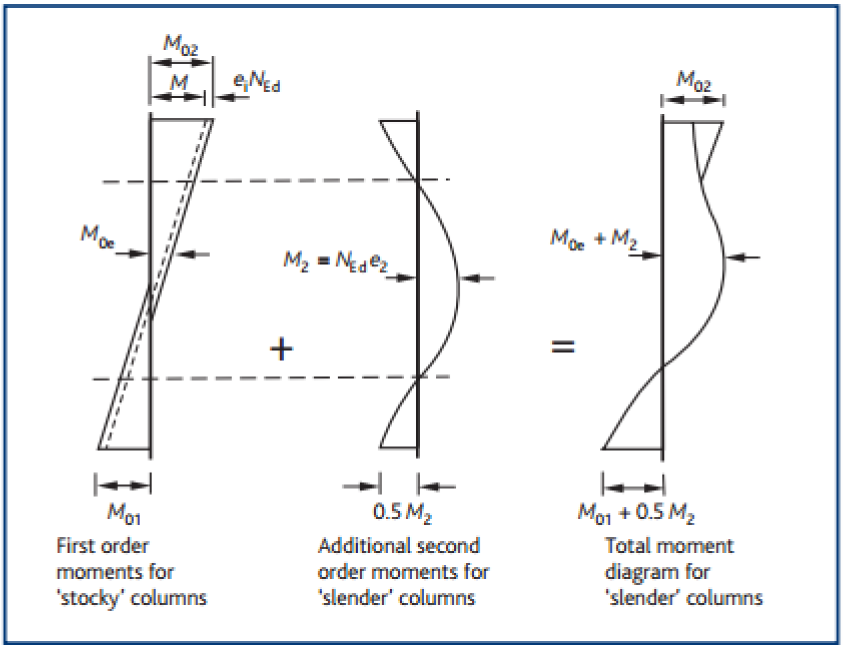
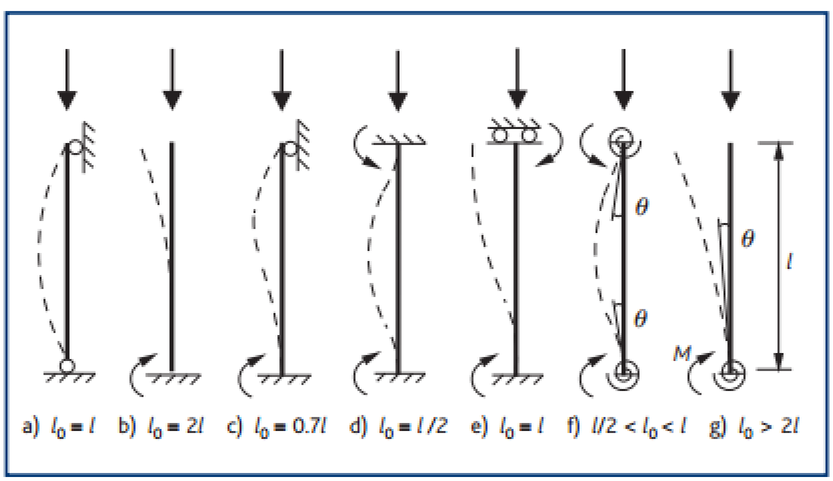


Fig 3. Effective Lengths for Isolated Members



Slenderness

Eurocode 2 states that second order effects may be ignored if they are less than 10% of the first order effects. As an alternative, if the slenderness (λ) is less than the slenderness limit (λ_{lim}), then second order effects may be ignored.

Slenderness, $\lambda = {}^{l_o}/_{I}$ where i= radius of gyration and slenderness limit.

$$\lambda_{lim} = \frac{20ABC}{\sqrt{n}} \leq \frac{15.4C}{\sqrt{n}} \tag{2.28}$$

Where

$A = 1/\left(1 + 0.2\varphi_{ef}\right)$ (If φ_{ef} is not known, A= 0.7 may be used) (2.29)

B= $\sqrt{1 + 2\omega}$, (if ω, reinforcement ratio, is not known, B= 1.1 may be used) (2.30)

C= 1.7 – r_m(if r_m is not known, C= 0.7 may be used (2.31)

$$n = N_{Ed}/(A_c f_{cd}) \tag{2.32}$$

$$r_m = M_{01}/M_{02} \tag{2.33}$$

If the end moments $M_{01}$and M_{02} give tension on the same side, r_m should be taken positive.

Of the three factors A, Band C, C will have the largest impact on λ_{lim} and is the simplest to calculate. An initial assessment of λ_{lim}can therefore be made using the default values for A and B, but including a calculation for C. Care should be taken in determining C because the sign of the moments makes a significant difference. For un members C should always be taken as 0.7.

Column design resistance

For practical purposes the rectangular stress block used for the design of beams may also be used for the design of columns fig below. However, the maximum compressive strain for concrete classes up to and including C50/60, when the whole section is in pure compression, is 0.00175(0.002 in EBCS). (See figure 5a). When the neutral axis falls outside the section (Figure 5b), the maximum allowable strain is assumed to lie between 0.00175 and 0.0035, and may be obtained by drawing a line from the point of zero strain through the 'hinge point' of 0.00175 strain at mid-depth of the section. When the neutral axis lies within the section depth then the maximum compressive strain is 0.0035.

Fig.4 Stress Block Diagram for Columns

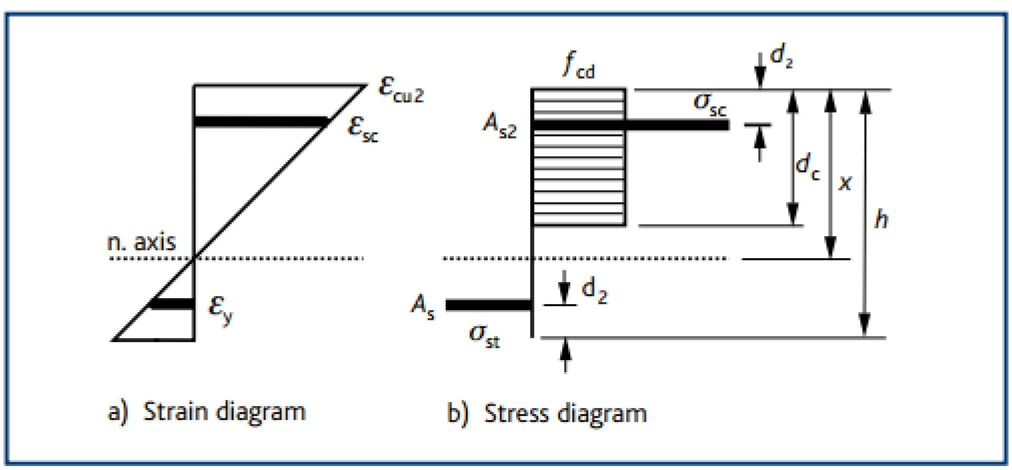
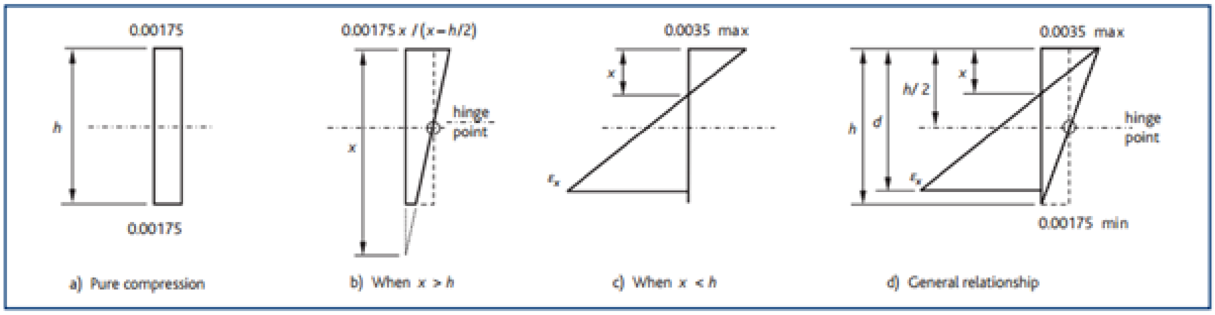


Fig.5 Strain Diagram for Columns



The general relationship is shown in Figure 5d. For concrete classes above C50/60 the principles are the same but the maximum strain values vary.

Two expressions can be derived for the area of steel required, (based on a rectangular stress block see Figure 4, one for the axial loads and the other for the moments:

$$A_{sN}/2 = (N_{Ed} - f_{cd}bd_c)/(\sigma_{sc} - \sigma_{st}) \qquad (2.34)$$

Where

A_{sN}= Area of reinforcement required to resist axial load

N_{Ed}= Axial load

f_{cd}= Design value of concrete compressive strength

$\sigma_{sc}(\sigma_{st})$ = Stress in compression (and tension) reinforcement

b= Breadth of section

d_c= Effective depth of concrete in compression = $\lambda x \leq h$

λ= 0.8 for ≤ C50/60

h= Height of section

$$A_{sM}/2 = [M - f_{cd}bd_c(h/2 - d_c/2)]/[(h/2 - d_2)(\sigma_{sc} + \sigma_{st})] \quad (2.35)$$

Where

A_{sM}= Total area of reinforcement required to resist moment

Realistically, these can only be solved iteratively and therefore either computer software or column design charts may be used.

2.1.2 Moment magnification method

(1) The total design moment, including second order moment, may be expressed as a magnification of the bending moments resulting from a linear analysis, namely:

$$M_{Ed} = M_{0Ed}\left[1 + \frac{\beta}{(\frac{N_b}{N_{Ed}}) - 1}\right] \quad (2.36)$$

Where:

M_{0Ed} is the first order moment;

β is a factor which depends on distribution of 1st and 2nd order moments,

N_{Ed} is the design value of axial load

N_B is the buckling load based on nominal stiffness

(2) For isolated members with constant cross section and axial load, the second order moment may normally be assumed to have a sine-shaped distribution. Then

$$\beta = \pi^2 / c_0 \tag{2.37}$$

Where:

c_0 is a coefficient which depends on the distribution of first order moment (for instance, c_0= 8 for a constant first order moment, c_0= 9.6 for a parabolic and 12 for a Symmetric triangular distribution etc.).

(3) For members without transverse load, differing first order end moments $M_{01}$and M_{02} may be replaced by an equivalent constant first order moment M_{0e} (see equation 2.5)

Consistent with the assumption of a constant first order moment, c_0= 8 should be used.

Note: The value of c_0= 8 also applies to members bent in double curvature. It should be noted that in some cases, depending on slenderness and axial force, the end moments(s) can be greater than the magnified equivalent moment.

(4) β= 1 is normally a reasonable simplification.

$$M_{Ed} = \frac{M_{0Ed}}{1 - (\frac{N_{Ed}}{N_B})} \tag{2.38}$$

Stress-strain relations of concrete for the design of cross sections

Fig.6a Parabola-Rectangle Diagram for Concrete under compression

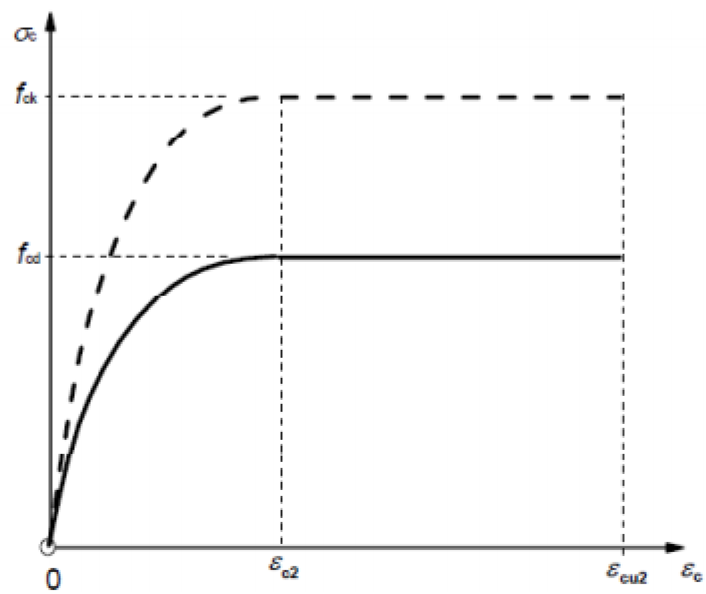
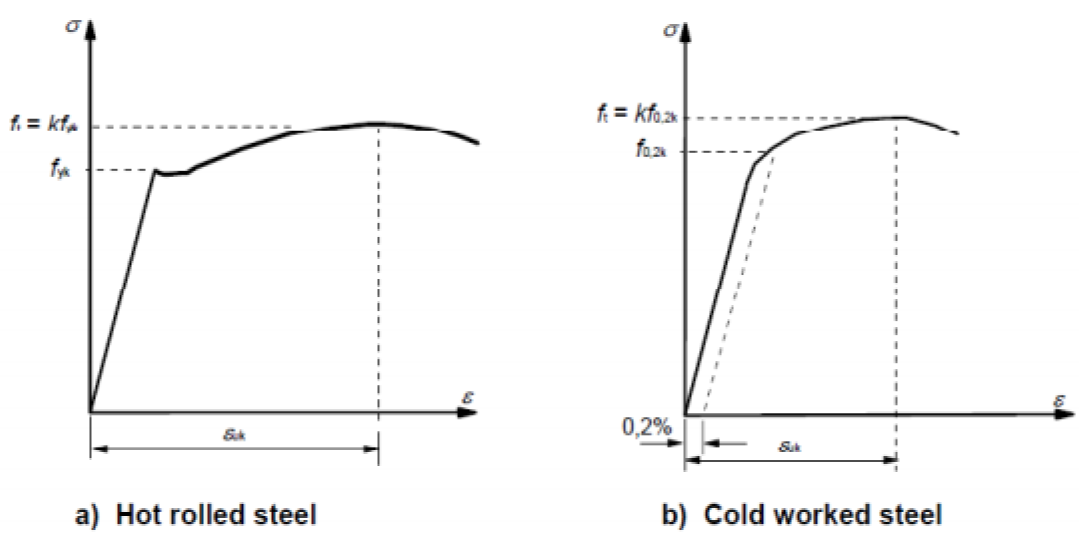


Fig 6b. Stress-strain diagrams of typical reinforcing steel



3. Method of the study

3.1 Basic assumptions in the analysis of sections in the ultimate limit states[2]

The method I used is based on the following assumptions which are widely used.

a) The plane sections remain plane after any deformation (Bernoulli's assumption).

b) Arbitrary monotonic stress–strain relationships for each of the three materials (i.e., concrete, structural steel and the reinforcing bars) may be assumed.

c) The longitudinal reinforcing bars are identical in diameter and are subjected to the same amount of strain as the adjacent concrete.

d) The maximum compressive strain in the concrete is taken to be:0.0035 in bending(simple or compound) 0.002 in axial compression,

e) The effect of creep and the tensile strength of concrete and any direct tension stresses due to shrinkage, etc., are ignored.

f) The maximum tensile strain in the reinforcement is taken to be 0.01.

g) Shear deformation is ignored

3.2 Project methodology

The values of (α_c, β_c) is derived as it is indicated in EBCS-2, Part_2,1995 in Case II and Case III in the following section. (λ & λ_{lim}) were taken from EUROCODE 2 .

[2] An improved design formula for a biaxial loaded slender RC column

Ultimate limit states for reinforced concrete sections

According to EBCS-2, a reinforced concrete section is in the ultimate limit state if the strain distribution over the cross-section lies in one of the five zones of strain profiles shown below.

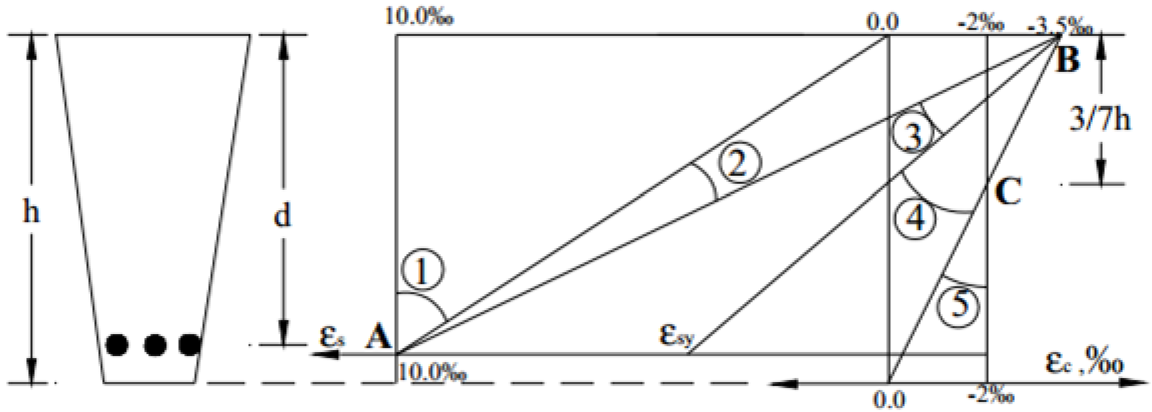


Fig.7 Strain distribution in the ultimate limit state [EBCS-2]

Strain profiles in zone (1) correspond to a situation where the whole cross section is subjected to tension. The three zones of strain profiles [(2),(3),(4)] have one common feature namely the neutral axis lies within the cross-section whereas zone (5) corresponds to a condition in which the whole cross-section is under compression and hence the neutral axis lies outside of the cross-section, a case of predominant compressive normal force with significant bending moment. In general there are various possibilities for the location of the neutral axis and each location corresponds to a particular combination of internal normal force and bending moments representing the ultimate capacity of the section.

Stress Resultant[3]

Each strain profile in the five zones corresponds to a particular combination of ultimate internal forces (normal force and bending moment) which are determined as the stress resultants in the concrete and reinforcement steel. The determination of stress resultants with the locations of the point of applications is one of the most important steps in the analysis of reinforced concrete sections and can broadly be divided in to two cases based on the location of the neutral axis.

Case- I: The neutral axis lies within the cross-section

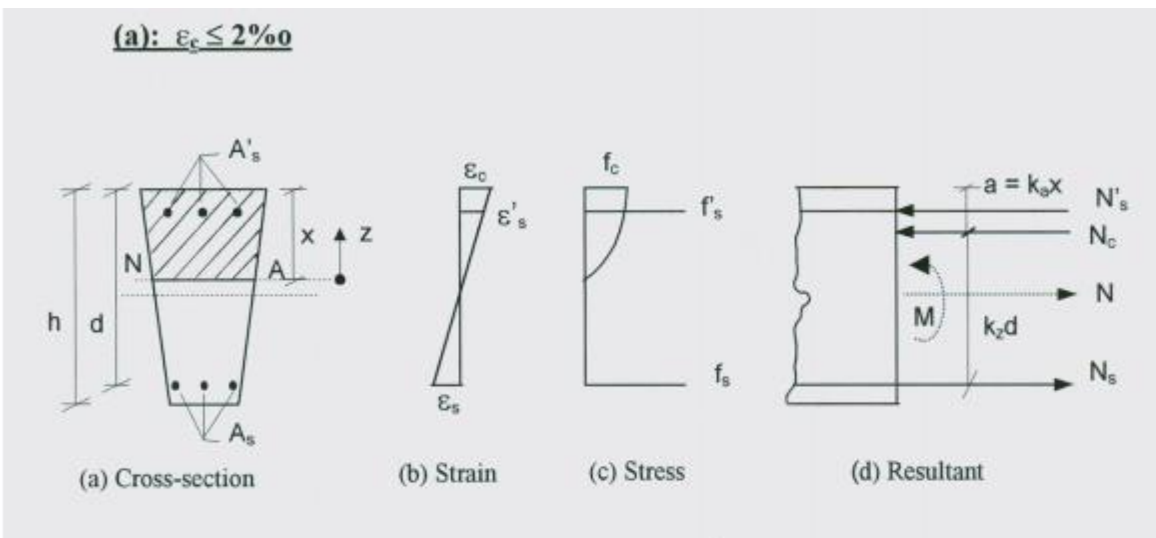


Fig.8 Strain Distribution case 1(a)

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1. [3] *Tefera Desta, may 1999,(EVALUATION OF APPROXIMATE METHODS FOR THE DESIGN OF BIAXIALLY LOADED REINFORCED CONCRETE COLUMNS) PAGE 5-11*

For the strain distribution shown in fig. (8, b) the strain at a distance z from the neutral axis can be expressed as:

$$\varepsilon = \frac{\varepsilon_c}{x} z$$

The stress in the concrete at the corresponding point can be determined from the idealized stress-strain relationship for concrete

$$f_c = \varepsilon\left(1 - \frac{\varepsilon}{4}\right) f_{cd} = \frac{\varepsilon_c}{x} z \left(1 - \frac{\varepsilon_c}{4x} z\right) f_{cd} \qquad (3.1)$$

The resultant normal force on the compressed concrete zone can be obtained by integrating the stress distribution over the compressed area.

$$\int_0^{z=x} f_c b_z dz \qquad (3.2)$$

Where b_z is the width of cross-section at distance z from the neutral axis.

Inserting equation (3.1) in to equation (3.2) and performing the integration over the given limits, the total compressive force on the compressed zone of concrete for sections with constant width b can be expressed as follows after rearranging terms.

$$N_c = \alpha_c b x f_{cd} \qquad (3.3)$$

Where:

$$\alpha_c = \frac{\varepsilon_c}{12}(6 - \varepsilon_c) \qquad (3.4)$$

The location of N_c from the most compressed edge, a, can be determined by finding the centroid of the stress distribution with reference to the same point.

$$a = x - \frac{1}{N_c}\int_0^{z=x} f_c\, b_z y dz \tag{3.5}$$

Substituting the value of N_c from equation (3.3) and the expression for f_c from equation (3.1) in to equation(3.5) and performing the integration over the given limits, the location of N_c from the most compressed edge for sections with constant width b can be expressed as follows:

Where:

$$a = k_a x \tag{3.6}$$

$$K_a = \frac{8-\varepsilon_c}{4(6-\varepsilon_c)} \tag{3.7}$$

(b) $\varepsilon_c > 2\%o$

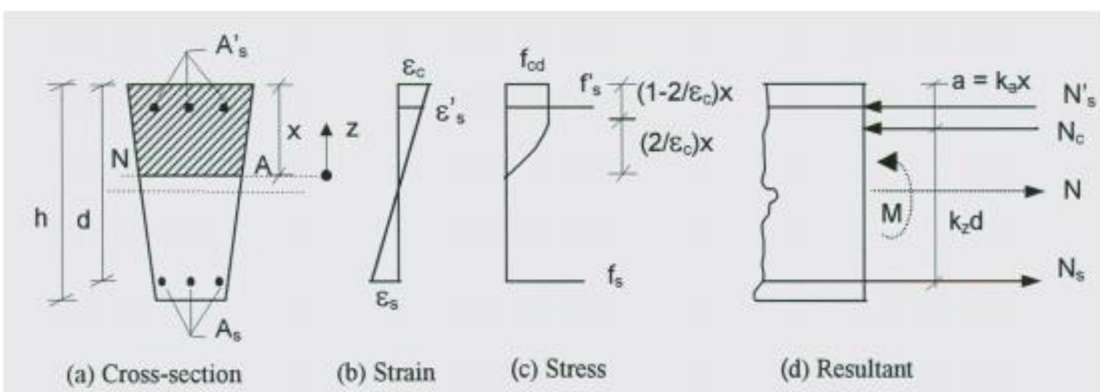


Fig.9 Strain distribution case 1(b)

In this case, the compressive stress distribution on concrete has two components: rectangular and parabolic, as shown in fig 9. The parabolic part can still be expressed by equation (3.1) and the resultant compressive force in the compression concrete area. For sections with constant width, this total force can be determined from the following:

$$N_c = b\left(1 - \frac{2}{\varepsilon_c}\right) x f_{cd} + b f_{cd} \int_0^{z=\frac{2x}{\varepsilon_c}} \frac{\varepsilon_c}{x} z \left(1 - \frac{\varepsilon_c}{4x} z\right) dz \qquad (3.8)$$

After performing the integration over the given limits and rearranging terms, N_c can be expressed with the same form of expression as equation (3.3) but different coefficient α_c as follows:

$$N_c = \alpha_c b x f_{cd} \qquad (3.9)$$

Where:

$$\alpha_c = \frac{3\varepsilon_c - 2}{3\varepsilon_c} \qquad (3.10)$$

The location of N_c from the most compressed fiber can be obtained using the same procedure as in the previous case. Thus the distance of the point of the compressive force in the concrete N_c from the outer most concrete fibers under compression is given by:

$$a = k_a x \qquad (3.11)$$

$$K_a = \frac{\varepsilon_c(3\varepsilon_c - 4) + 2}{2\varepsilon_c(3\varepsilon_c - 2)} \qquad (3.12)$$

Case-II the neutral axis lies outside the cross-section.

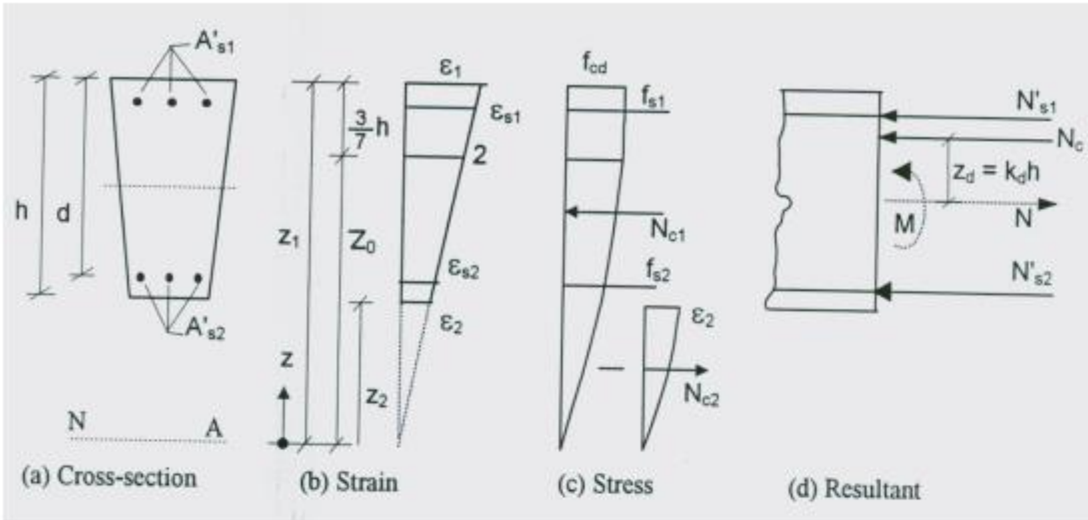


Fig.10 Strain distribution case 2

In this case the whole cross-section is under compression and the neutral axis lies outside of the cross-section. The strain distribution for this case is such that a fiber located at 3h/7 from the most compressed edge has a strain level of 2‰ as shown above in fig. 10. With this strain distribution, the strain at the less compressed edge ε_2, its distance from the neutral axis Z_2, the corresponding distances z_1 and z_0, of the most compressed edge and the concrete fiber with strain level equal to 2‰ respectively, can be determined from the stress distribution shown in fig above (b). They are given by:

$$\varepsilon_2 = \frac{14-4\varepsilon_1}{3} \quad (3.13)$$

$$z_1 = \frac{3\varepsilon_1}{7\varepsilon_1 - 2} h \quad (3.14)$$

$$z_2 = \frac{3\varepsilon_1}{7\varepsilon_1 - 2} h \quad (3.15)$$

$$z_0 = \frac{6}{7(\varepsilon_1 - 2)} h \tag{3.16}$$

The stress resultant Nc can be determined by integrating the compressive stress over the whole cross-section.

$$N_c = N_{c1} - N_{c2} = \int_0^{z=z_1} f_c\, b_z dy - \int_0^{z=z_2} f_c\, b_z dz \tag{3.17}$$

By performing the integration in the same way as in the previous cases, it can be shown that the resultant compressive force on the concrete for a section with constant width b can be expressed as follows:

$$N_c = \alpha_d b h f_{cd} \quad \text{Tentative} \tag{3.18}$$

Where:

$$\alpha_d = \frac{1}{189}(125 + 64\varepsilon_1 - 16\varepsilon_1^2) \tag{3.19}$$

The location of Nc, in this case from the centroid of the cross-section, can be determined with similar procedure as in the previous cases and found to be as follows:

$$Z_d = k_a h \tag{3.20}$$

$$K_d = \frac{40}{7}\left[\frac{(\varepsilon_1 - 2)^2}{125 + 64\varepsilon_{1\_} - 16\varepsilon_1^2}\right] \tag{3.21}$$

After determining the total compressive force on concrete and its point of application; the next step is to compute the forces in the reinforcement steel, which are easily determined by using the strain distribution diagram and idealized stress-strain relationship for steel.

Once the internal forces in the steel and concrete with their locations are calculated, the capacity of the given cross-section at ultimate limit states can be determined by summing up all the forces in the axial direction and their bending moments about the centroid of the cross-section.

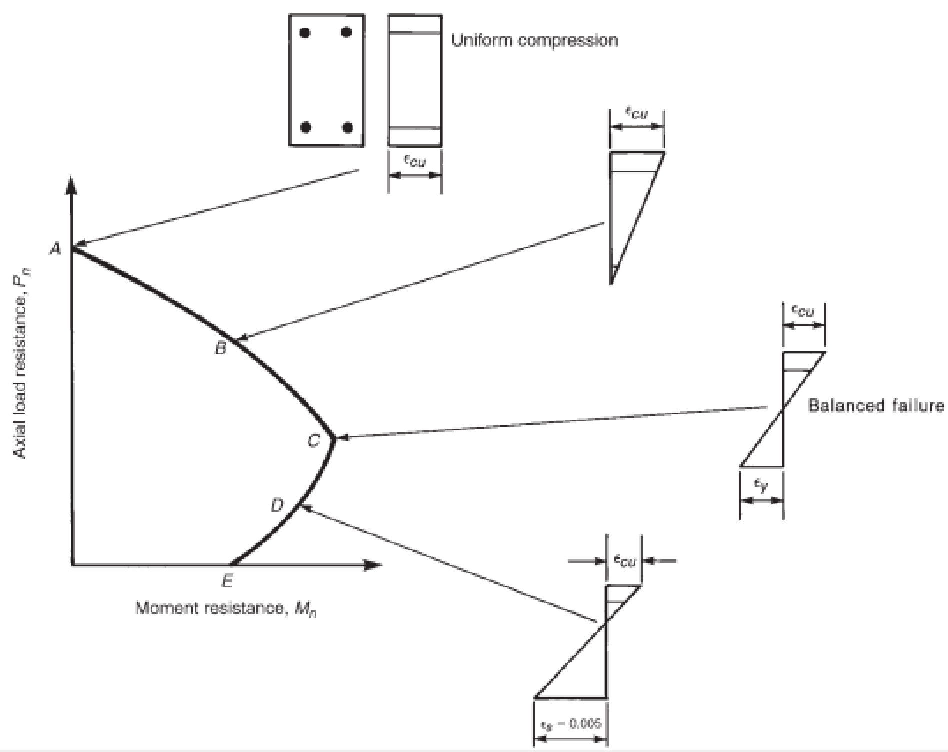
Compression with Uniaxial Bending

Almost all compression members in concrete structures are subjected to moments in Addition to axial loads. These may be due to the load not being centered on the column, or may result from the column resisting a portion of the unbalanced moments at the ends of the beams supported by the column. Since reinforced concrete columns are normally framed in two orthogonal directions, they are subjected to biaxial bending. Uni-axial bending is a special case where eccentricity of the loading is along one or the other axis and the neutral axis is therefore parallel to the axis of bending.

The capacity of cross-sections under compression with uniaxial bending at the ultimate limit state is best expressed by plotting the axial load N-versus the bending moment M at failure called N-M interaction diagram. Interaction diagrams for reinforced concrete cross sections can in general be plotted by assuming a series of strain distribution at failure in different zones of the strain profile shown in fig. 7 and computing the corresponding values of N and M. One pair of such values represents the coordinates of a particular point on the interaction diagram.

In this approach, Tefera (1999: *5-11*) noted that the strains could be varied in a controlled manner, however, the corresponding points on the interaction diagram cannot be expected to be evenly spaced because the internal forces are not linearly related to strain. The points could therefore be clustered together at some regions while dispersed far apart at other regions, which make it unsuitable for systematic generation of interaction charts. A preferred procedure would therefore be to vary the normal force N in a controlled manner and determine the associated ultimate bending moment; a much more difficult task requiring iteration.

The points so obtained are evenly spaced and result in a smooth or best-fit plot of the interaction diagram.



Source: James G. MacGregor and James K.Wight, 2005. Reinforced Concrete Mechanics and Design, Fourth edition, New Jersey

Fig. 11 strain distribution over the cross-section and corresponding point on the interaction diagram.

3.3 Conjugate beam method[4]

The conjugate beam method is an extremely versatile method for computation of deflections in beams or columns; generally, the relationships between the loading, shear, and bending moments are given by

$$\frac{d^2M}{dx^2} = \frac{dV}{dx} = -w(x) \tag{3.22}$$

Where M is the bending moment; V is the shear; and w (x) is the intensity of distributed load.

Similarly, we have the following

$$\Phi = \frac{d^2v}{dx^2} = \frac{d\theta}{dx} = \frac{M}{EI} \tag{3.23}$$

For this research, I've determined curvature directly from strain profile.

$$\Phi = \frac{\epsilon_{ct} - \epsilon_{cb}}{h} \tag{3.24}$$

Where

ϵ_{ct}= strain on top fiber of section

ϵ_{ct}=strain on bottom fiber of section

h=depth of section

[4] [http://nptel.ac.in/courses/105101085/downloads/lec-23.pdf]

A comparison of two set of equations indicates that if Φ is the loading on an imaginary beam, the resulting shear and moment in the beam are the slope and displacement of the real column, respectively. The imaginary beam is called as the “**conjugate beam**” and has the same length as the original beam.

There are two major steps in the conjugate beam method. The first step is to set up an additional beam, called "conjugate beam,” and the second step is to determine the “Shearing forces” and “bending moments” in the conjugate beam.

For each existing support condition of the actual beam, there is a corresponding support condition for the conjugate beam. Table 3.1 below shows the corresponding conjugate beam of different types of actual beams. The actual beam as well as the conjugate beam is always in static equilibrium condition.

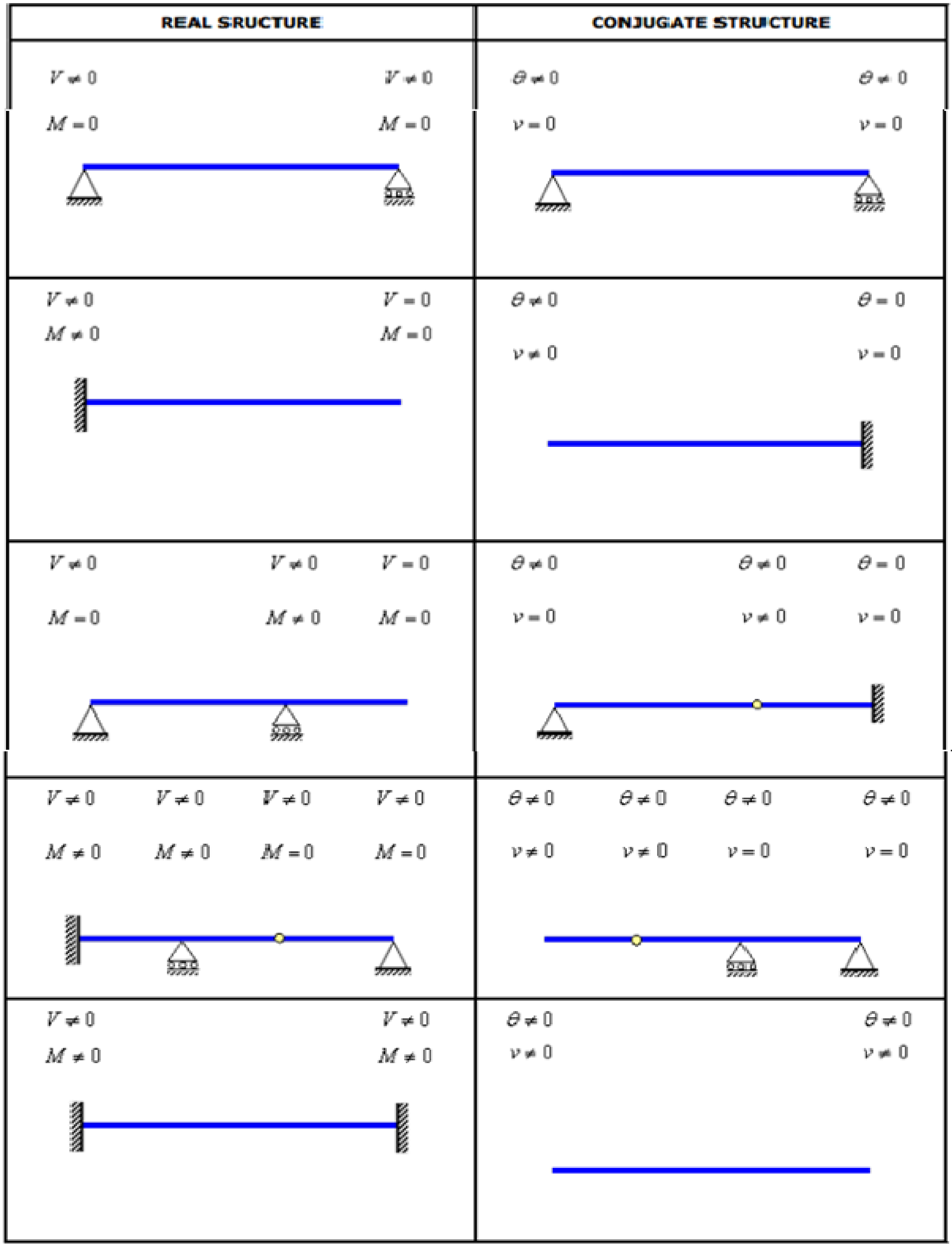


Table 3.1 Real and Conjugate beams for Different structures

For the process, a program is used (C++)

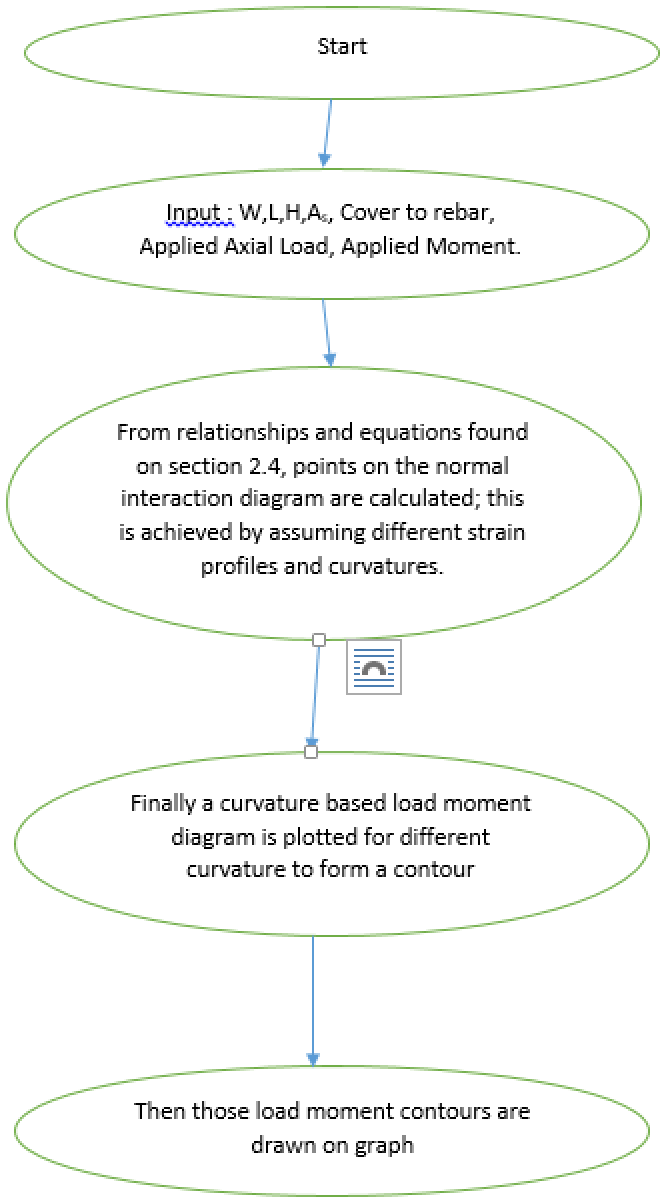


Fig.12 Flow Chart of Program

4. Findings of the study

In the beginning of the study, I used the integration method for determining deflection. This is integrating M/EI equation over the column, and repeating the process until convergence was obtained. This procedure would have been accurate if crack effects were ignored. But in reality, cracks are common parts of columns, hence I had to include them as factors during the study. I considered two types of sections, a normal cross-section un-cracked and a fully cracked section (This was accommodated by equation 3.24.). In a result, the deflection outcome was farther to the ones in ETABS software yielded.

For this very reason I had to rethink my calculation and use a different approach. I used first order moment and axial load. I used C++ program to yield a different value of curvature.

5. Conclusion and Recommendations

5.1 Conclusion

Researchers have made several improvements for the design of slender column over the years. The transition from reduction factor method to moment magnification procedure was a major step. An attempt was made to simplify the design time (computational savings) over the method outlined in the ACI design handbook and the regression formulas developed as an alternative to the ACI methods are some examples.

In this study the approximate interaction diagram for slender RC column for different heights is presented. A simple and user-friendly program was developed to determine the moment-load contours based on assumed curvature.

The essential steps of the procedure can be summarized as follows:

1. Starting from geometric, reinforcement and material data (b,h,L,A_s,f_{ck},f_{yk}) the unit less load and moment data (μ_{input}& v_{input}) are obtained. (the column will be divided in to different segments)
2. From relationships and equations found on section 2.4, points on the normal interaction diagram are calculated; this is achieved by assuming different strain profiles and curvatures.
3. Finally a curvature based load moment diagram is plotted for different curvature to form a contour.

5.2 Recommendations

Finally, the following recommendations are made.

1. In this study, only rectangular cross-sections were considered. Other sections such as circle or triangular can be used for future studies.
2. The focus of the research was isolated column, columns with other end conditions can be used for future studies.
3. Bi-axially loaded slender columns can be the subject of future studies.
4. The program can be refined to save more computational time.

4. Reference

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Hyo-Gyoung Kwak,Ji-Hyun Kwak, *2009 (an improved design formula for a biaxially loaded slender RC column, Department of civil and environmental engineering, Korea advanced institute of science and technology, Daejeon 305-701, republic of Korea, page 236)*

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Moss R and Brooker O, *The Concrete Centre™ and British Cement Association*, 2006.

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Annex 1: Detail Calculation

Step 1: Determine material property

Cross-section property:

Height of column=400mm;

Cover of concrete= 35mm;

Width of concrete=200mm;

As=2400mm^2;

Fcd= 13.6;

Step 2: Determine top and bottom strain from curvature data.

Curvature points are assumed.

Ect(top concrete strain) are assumed.

Curvature = 0.002

Ecb=0.7

*Ecb(bottom strain concrete)= Ect-(curvature * d)*

*Ecb= 0.7-(0.002*365)*

Ecb=-0.03

Step 3: Determine the neutral axis position (X)

*X =(Ect*H)/(-Ecb+Ect)*

*X= (0.7*400)/(0.03+0.7)*

X= 280/0.73= 383.5616mm

Step 4: Determine Top strain for steel(Est)

Est=(Ect/X)(X-cover)*

=(0.7/383.561)(383.561-35)*

=(0.001825)*(348.561)

=0.636123825

Step 5: Determine bottom strain for steel (Esb)

Esb=(-Ect*((H-cover)-X))/X

Esb=-0.7*((400-35)-383.561)/383.561

Esb=(-0.7*-18.561)/383.561

Esb=0.03387

Step 6: Determine stress

Fst (stress in top reinforcement steel)=E*(Est*0.001)

= 200,000*(0.636*0.001)

=127.2

Fsb(stress in bottom reinforcement steel)=E*(Esb*0.001)

Fsb= 200,000*(0.03387*0.001)

Fsb=6.774

Step 7: determine concentrated force

Cst(concentrated force located in top reinforcement steel)=Fst*As*0.5

Cst=127.2*2400*0.5

Cst= 152,640

Tsb(tension force located in bottom reinforcement steel)=Fsb*As*0.5

=6.774*2400*0.5

=8,128.8

Cc(concrete compression force)= Alpha*Fcd*B*d

Since Ect<2 and X<400

$Alpha= (Ect*0.08333)*(6-Ect)*Kx$

$Alpha= (0.7*0.08333)*(6-0.7)*(383.561/365)$

$Alpha=0.324$

$Cc=0.324*13.6*200*365$

$Cc=321{,}667.2$

$Pnmax=Cst+Tsb+Cc$

$Pnmax=152{,}640+8128.8+321667.2$

$Pnmax=482{,}436$

$Vu=Pnmax/(Fcd*B*H)$

$Vu=482{,}436/(13.6*200*400)$

$Vu=0.4434$

$Bc=(0.25*(8-Ect)/((6-Ect)))*Kx$

$Bc=(0.25*(8-0.7)/(6-0.7))*(383.561/365)$

$Bc=0.362$

$Mu=Cc*(H*0.5-Bc*d)+Cst*(H*0.5-h)-Tsb*(H*0.5-h)$

$Mu=321{,}667.2*(400*0.5-0.362*365)+152{,}640*(400*0.5-35)-$

$8128.8*(400*0.5-35)$

$Mu=21831552.86+25185600-1341252$

$Mu=45{,}675{,}900.86Nmm$

$Mew=Mu/(Fcd*B*H^2)$

$Mew=45675{,}900.86/(13.6*200*400^2)$

$Mew=0.104953$

The program then repeats this process until enough points have been found for plotting.

Annex 2: Code

```
// ConsoleApplication6.cpp : Defines the entry point for the console application.
//curvature program//

#include "stdafx.h"
#include <iostream>
#include <cmath>
#include <string>
#include <fstream>
using namespace std;
double Ect, Ecb, Est, Esb, H, X, h, d, Fsb, Fyd, Tsb,Cc, Alpha, Fcd, B, Pnmax, Vu,
Mu,
Cs2, Cs1, Bc, Mew, curvature,Kx;
float  Cst, Fst, As, E;
int N, i, F;


int main()
{
	ofstream outputFile;

	outputFile.open("for Dr Esayas.txt");


	for (curvature = 0.002;curvature <= 0.0369;curvature += 0.002) //curvature
loop//
	{
		Ect = 0.1;
		H = 400;
		h = 35;
		B = 200;
		d = 365;
		E = 200000;
		As = 2400;
		Fcd = 13.6;
		do
		{
			//strain loop//
			Ecb = Ect - (curvature*d);
			if(Ecb<0)
			X = Ect*H / (-Ecb + Ect);//negative sign because means strain
negative(tension)//
			if(Ecb>0)
			X = Ect*H / (Ecb + Ect);
			Est = (Ect / X)*(X - h); //Est is always positive because X is
greater than cover depth//
			Esb = -Ect*((H - h) - X) / X;

			if ((Est<2) & (Est>-2))
```

```
                                Fst = E*(Est*0.001);

                        if ((Esb<2) & (Esb>-2))
                                Fsb = E*(Esb*0.001);
                        cout << "Fst=" << Fst << " " << "Fsb=" << Fsb << endl;
                        Kx = X / d;
                        cout << "Kx=" << Kx << endl;
                        if((Ect<=2) & (X<400))
                        {
                                Alpha = (Ect*0.08333)*(6 - Ect)*Kx;
                                Bc = (0.25*(8 - Ect) / ((6 - Ect)))*Kx;
                                cout << "Alpha=" << Alpha << " " << "Bc=" << Bc <<
endl;
                        }

                        if ((Ect > 2) & (X < 400))
                        {
                                Alpha = (0.333*(3 * Ect - 2) / Ect)*Kx;
                                Bc = (0.5*(Ect*(3 * Ect - 4) + 2) / (Ect*(3 * Ect -
2)))*Kx;

                                cout << "Alpha=" << Alpha << " " << "Bc=" << Bc <<
endl;
                        }
                        if((Ect>2) & (X>400))
                        {
                                Alpha = (0.005291)*(125 + 64 * Ect - 16 * pow(Ect, 2));
                                Bc = 0.5 - (5.714)*(pow(Ect - 2, 2) / (125 + 64 * Ect -
16 * pow(Ect, 2)));

                                cout << "Alpha=" << Alpha << " " << "Bc=" << Bc <<
endl;
                        }

                        if (Est >= 2)
                                Fst = Fyd;
                        if (Est <= -2)
                                Fst = -Fyd;
                        if (Esb >= 2)
                                Fsb = Fyd;
                        if (Esb <= -2)
                                Fsb = -Fyd;
                        cout << "Fst=" << Fst << "Fsb=" << Fsb << endl;
                        Cst = Fst*As*0.5;
                        cout << "Cst=" << Cst << endl;
                        Tsb = Fsb*As*0.5;
                        cout << "Tsb=" << Tsb << endl;
                        Cc = Alpha*Fcd*B*d;
                        cout << "Cc=" << Cc << endl;
                        Pnmax = Cst + Tsb + Cc; //because the column is under
compression//

                        cout << "Pnmax=" << Pnmax << endl;
                        Vu = Pnmax / (Fcd*B*H);
                        cout << "Vu=" << Vu << endl;
                        Mu = Cc*(H*0.5 - Bc*d) + Cst*(H*0.5 - h) - Tsb*(H*0.5 - h);
                        cout << "Mu=" << Mu << endl;
                        Mew = Mu / (Fcd*B*pow(H, 2));
                        cout << "Mew=" << Mew << endl;
                        //outputFile << Vu << " " << Mew << endl;
                        //i may need to put some conditions on filtering the results//
```

```
                    outputFile << "curvature= " << curvature << endl;
                    outputFile << "Cst= "<<Cst << " " << "Tsb= "<<Tsb << endl;
                    outputFile << "Cc= " << Cc << endl;
                    outputFile << Vu << " " << Mew << endl;
                    cout << "Ect= " << Ect << endl;
                    cout << "curvature= " << curvature << endl;
                    cout << "Cst= " << Cst << " " << "Tsb= " << Tsb << endl;
                    cout << "Cc= " << Cc << endl;
                    cout << "Vu= " << " " << Vu << " " << "Mew= " << Mew << endl
<< endl;
                    Ect = Ect + 0.1;

             } while (Ect <= 3.5);
             outputFile << "" << endl;
       }
       cin >> F;
    return 0;
       }
```